

Planning and optimization of intermodal hub-and-spoke network under mixed uncertainty



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ABSTRACT

This paper addresses the planning and optimization of intermodal hub-and-spoke (IH&S) network considering mixed uncertainties in both transportation cost and travel time. Different from previous studies, this paper develops a novel modeling framework for the IH&S network design problem to jointly minimize the expected value of total transportation costs and the maximum travel time requirement in term of critical value. A new hybrid methodology by combining fuzzy random simulation (FRS) technique and multi-start simulated annealing (MSA) algorithm is designed to solve the proposed model. Numerical experiments are implemented to verify the effectiveness of the proposed model and solution approach.

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1. Introduction

Intermodal transportation involving two or more modes of transportation provides an economical solution for the cargo delivery. Intermodal transportation differs from multimodal transportation, where the latter refers to the choice of a single mode of transportation among the available modes. The intermodal transportation is already being used by the package carrier United Parcel Service (UPS), which started as a package carrier using air transportation, has added road and rail shipments to its operations. Their use of road/rail/air has expanded to an extent that UPS is now the largest US customer of intermodal services. As suggested by Crainic and Kim (2007), an intermodal transportation system can be essentially formulated as an intermodal hub-and-spoke (IH&S) network. In addition to locating hubs and allocating origin and destination (O-D) nodes to hubs, as addressed in conventional hub-and-spoke (H&S) network design (Gelareh et al., 2015; Hsiao and Hansen, 2011; Ishfaq and Sox, 2010), the IH&S network design needs to identify optimal mode-change transshipment lines at hubs.

In an IH&S network design, there are usually two kinds of operational performance metrics that need to be considered, i.e., transportation cost and travel time. In this research, total transportation costs and maximum travel time requirement are used as the performance criteria. More specifically, the total transportation costs is related to the benefit of company, and the maximum travel time requirement is referred to evaluate the service quality for customers. Most of the previous studies take into account the total transportation costs or the maximum travel time requirement independently. The motivation for our work stems from our view that, models that combine both dimensions could provide valuable insights for designing IH&S network. To this aim, a desirable IH&S network configuration is required to achieve the minimization of both total transportation costs and maximum travel time requirement.

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Furthermore, designing the IH&S network is a long term strategic planning problem which can be affected by uncertainties. That is, the available data/possible values of the transportation cost and travel time are hardly deterministic or known too much in advance, leading to the need of considering uncertainty. Some uncertainties can be described as fuzzy variables or random variables. Noteworthy, fuzzy data are mainly generated by relying on subjective/judgmental opinions, experiences and feelings of field experts, while random data are estimated based upon the availability of sufficient objective data (i.e., available historical data). In fact, due to the imprecise human knowledge and subjective judgement in capturing historical data, uncertain variable may include fuzziness and randomness simultaneously. To tackle this hybrid uncertainty, fuzzy random variable (Kwakernaak, 1978) may be an appropriate research tool in capturing the mixed uncertainty associated with the transportation cost and travel time. This research focuses on the planning and optimization of an IH&S network under mixed uncertainty.

In order to conduct the above mentioned analysis, this research develops a novel modeling framework and a solution methodology. Extending the conventional H&S network design (HSND) approach to the intermodal transportation domain, the modeling framework accommodates the operational structure of individual modes of transportation and the interaction between modes, total transportation cost and travel time requirements. It also uses two type of decision criteria to systematically characterize the efficiency (low transportation cost) and effectiveness (high service quality) of IH&S network. Specifically, the expectation criterion is to evaluate the financial concern. The critical value criterion is to quantify the service concern. These two decision criteria can address two fundamental questions in IH&S network planning and optimizing, i.e., “how much money are there for the company to operate the IH&S network?” and “how fast a service will be provided for customers in IH&S network?” To implement these two decision criteria in practice, we propose a fuzzy random intermodal hub-and-spoke network design (FR-IHSND) problem to evaluate the network performance. From this base, we get an insight on how relevant a modeling framework comprising uncertainty aspects may be in IH&S network design problems. In earlier research, solution techniques used to solve IH&S network problems had been restricted to the iterative application of the shortest path method and/or local search heuristics. Those techniques can be improved through the use of meta-heuristics which have shown great promise in the realm of large-scale networks. In this research, a hybrid multi-start simulated annealing (HMSA) algorithm which incorporates fuzzy random simulation (FRS) technique with multi-start simulated annealing (MSA) algorithm is developed to solve the FR-IHSND problem. Specifically, FRS is designed to calculate the uncertain functions in this paper, and MSA combines the advantages of the simulated annealing algorithm with the multi-start strategy. The HMSA and a hybrid genetic algorithm (HGA) are compared to assess the effectiveness of the proposed solution method.

The remainder of this paper is organized as follows. Section 2 presents previous research in the conventional H&S network domain and identifies the contributions of this work. Section 3 describes a modeling framework and a mathematical model for the IH&S network design problem under mixed uncertainty. In Section 4, we derive the crisp counterpart model for the proposed problem in some special cases. The technique of fuzzy random simulation are presented in Section 5. After that, a hybrid intelligent algorithm, which incorporates FRS method and MSA algorithm, is designed to solve the general model in Section 6. In Section 7, some numerical experiments are conducted to demonstrate the effectiveness of the proposed model and solution approach. Finally, Section 8 provides conclusions and suggestions for future studies.

2. Literature review

This section presents a review of the H&S network literature that provides the background and establishes a framework for this research. The research presented in this paper draws from the concepts in the literature related to the conventional HSND problem and developing solution algorithms. This section also identifies the contributions of the research presented in this paper.

The conventional HSND problem has been extensively investigated in past studies (Alumur and Kara, 2008; Farahani et al., 2013), which mainly contains hub median problem and hub center problem. O’Kelly (1986, 1987) provided a quadratic integer programming formulation for hub median problem. Skorin-Kapov et al. (1996) developed tight linear relaxations of the formulation for hub median problem. Campbell (1994) proposed the first formulation for hub center problem as a quadratic programming model. Kara and Tansel (2000) provided several linear formulations for hub center problem. New formulations of the HSND with fewer variables and constraints were developed by Ernst and Krishnamoorthy (1998). In addition to locating hubs and allocating O-D nodes to hubs, as addressed in the conventional HSND problem, this research incorporates the choice of a transportation mode over inter-hub links.

In the previous literature, researchers have studied the conventional HSND problem under stochastic uncertainty. For example, Yang (2009) presented a two-stage stochastic programming model for air freight hub location and flight route planning under seasonal demand variations. Sim et al. (2009) attempted to tackle HSND with stochastic travel times involving mutually independent normal distributions. Contreras et al. (2011) studied stochastic uncapacitated HSND in which uncertainty is associated to transportation costs. Mohammadi et al. (2013) proposed a new stochastic multi-objective transportation model for HSND problem under stochastic uncertainty in travel times. Although these models extend the conventional HSND problem to random situations, they do not jointly consider uncertainties in both transportation cost and travel time.

Another aspect of HSND problem attempts to consider the fuzzy uncertainty. For instance, Chou (2010) proposed a fuzzy multiple criteria decision-making model for evaluating and selecting the container transshipment hub port. Taghipourian

et al. (2012) presented a fuzzy integer linear programming approach to dynamic virtual HSND with the aim of minimizing the transportation cost in a network. Yang et al. (2013) presented a new risk aversion HSND with fuzzy travel times by adopting value-at-risk criterion in the formulation of the objection function. Mohammadi and Moghaddama (2016) proposed a fuzzy HSND problem by incorporating a fuzzy M/M/1 queuing system. However, in practical HSND, uncertainty may present both fuzziness and randomness. To the best of our knowledge, there are only two studies dealing with the HSND taking the mixed uncertainty into account. Mohammadi et al. (2014) proposed a mixed possibilistic-stochastic programming model for the HSND. Yang and Liu (2015) develops three new equilibrium optimization models for HSND problem, in which the travel times are characterized by fuzzy random variables. Each of these two models is a straightforward extension of the fuzzy HSND problem. However, these two studies can not integrate financial and service considerations into the models to reflect the interplay between the financial and service issues.

Compared to the conventional HSND problem, the IHSND problem has received limited attention, presumably because intermodal transportation is a recently emerging research field and has not been so far explored in detail (Bontekoning et al., 2004). A few studies have been carried out for addressing the problem. Arnold et al. (2004) formulated an integer programming model for the IHSND problem. For instance, Racunica and Wynter (2005) gave an optimal hub location model aiming to increase the market share of rail mode in a H&S network. Meng and Wang (2011) proposed a mathematical formulation to design an intermodal hub network for multi-type container transportation. Although these models extend the conventional HSND problem to involve partial characteristics of intermodal transportation operations, they neglect the multi-criteria decision making nature of IHSND problem. Moreover, these models are inadequate to formulate the IHSND problem under mixed uncertainty, which is widely seen in practical intermodal transportation.

Another important aspect in HSND is how to solve them effectively, since they are known to be NP-hard (Alumur and Kara, 2008). In literature, various heuristics based approaches have been used extensively to solve HSND. For example, Chen (2007) proposed two approaches to determine the upper bound for the number of hubs along with a hybrid heuristic based on the simulated annealing method, tabu list, and improvement procedures to solve the proposed uncapacitated HSND. Azizi et al. (2016) proposed a heuristic based on genetic algorithm to solve the problem of configuring H&S networks under the risk of hub disruption. Randall (2008) applied ant colony optimization in order to solve a capacitated single allocation HSND. Calik et al. (2009) presented a tabu search heuristic for the HSND. Saboury et al. (2013) proposed two hybrid heuristics algorithms which incorporated a variable neighborhood search algorithm into the framework of simulated annealing and tabu search. Martí et al. (2015) presented a scatter search implementation for an NP-hard variant of the classic HSND. Among these solution approaches, simulated annealing has been very successful in finding close-to-optimal solutions for large size problems. To the best of our knowledge, the conventional SA algorithm is slow in convergence, and the implementation of SA algorithms is problem-dependent. Therefore, many scholars have been researching on improving the conventional SA algorithm (Azizi and Zolfaghari, 2004; El-Bouri et al., 2007; Yu and Lin, 2015). To overcome such intrinsic limitations, this paper presents a hybrid multi-start simulated annealing (HMSA) algorithm which incorporates the fuzzy random simulation (FRS) technique with multi-start simulated annealing (MSA) algorithm.

3. Problem description and formulation

In this research an IH&S network is represented by a graph in which nodes represent demand points and arcs represent different modes of transportation between the nodes. In the context of this research, an hub has local access to road, rail and air freight terminals. A shipment may be sent between hubs through the use of any one of the available modes. The choice of modes (road, rail, or air) for moving a shipment between hubs is determined by the tradeoff between the transportation cost associated with each mode and the maximum travel time requirement quoted to the customer. Fig. 1 shows the different

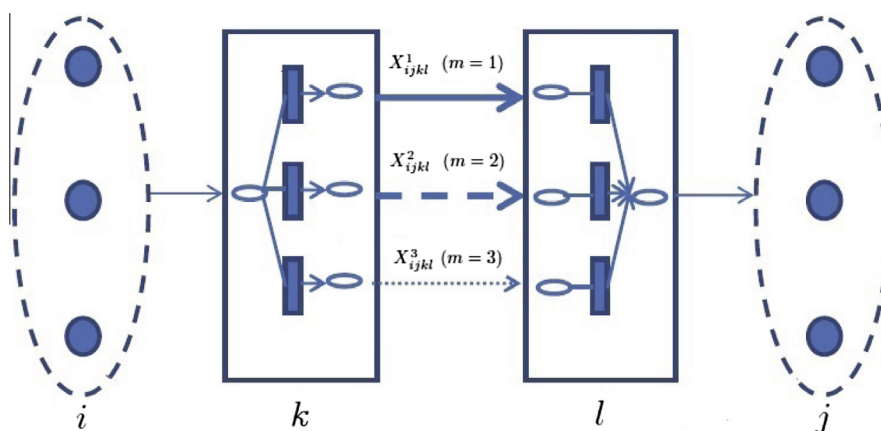


Fig. 1. Types of shipments in intermodal network.

ways that a shipment may travel between its origin and its destination. The shipment may also travel through the hubs k and l using transportation mode m , which is represented by the binary variable X_{ijkl}^m . Every hub shipment travels from its origin city i to its origin hub k using road transportation. The travel between the origin hub k and the destination hub l may occur over any one of the arcs which represent the different modes of transportation between the hubs. The choice of a specific arc represents the use of the corresponding transportation mode. Although a specific mode is selected for an inter-hub shipment for a specific O-D pair, other O-D pairs may flow through the same hubs using a different mode of transportation. Thus, for a specific O-D pair (i, j) and hub pair (k, l) , X_{ijkl}^1 implies road travel mode between the hub cities, X_{ijkl}^2 implies rail travel mode, and X_{ijkl}^3 implies air travel mode. For each O-D pair, only one mode is selected. The final leg of travel from the destination hub k to the destination city j always uses road transportation.

In this section, we present a mathematical formulation for the IHSND problem under mixed uncertainty. We consider a simple base situation in order to get a more focused demonstration of the proposed methodology. Nevertheless, it should be noted that the contents of the following sections can be extended to more complex IHSND models than those we address.

3.1. Fuzzy random information representation

In real-world applications, stochastic variability (randomness) and vagueness or fuzziness may coexist in the IHSND problem. On one hand, due to the subjective judgement and imprecise human knowledge and perception in capturing statistic data, the parameters of the real IHSND problem may embrace randomness and fuzziness at the same time. On the other hand, sometimes the historical data available for the parameter-distributions in IHSND problem are insufficient. Therefore, the expert knowledge (fuzzy information) should be incorporated into the available statistical data. In both cases mentioned above, there is a genuine need to deal with a hybrid uncertainty of randomness and fuzziness in IHSND problem. To characterize uncertain information, this paper will then represent the transportation cost and travel time as fuzzy random variables (vectors) by both credibility and probability distributions. For the completeness of this research, we shall briefly introduce some relevant concepts and results in fuzzy random theory in the following discussion.

Let Γ be an abstract space of generic elements. An ample field \mathcal{A} on Γ is a class of subsets of Γ that is closed under arbitrary unions, intersections, and complement in Γ , and Pos is a possibility measure defined on \mathcal{A} . A self-dual set function Cr, called credibility measure (Liu and Liu, 2002), was defined as follows

$$Cr(A) = \frac{1}{2}(1 + Pos(A) - Pos(A^c)), \quad A \in \mathcal{A},$$

where $A^c = \Gamma \setminus A$. The triplet $(\Gamma, \mathcal{A}, Cr)$ is called a credibility space.

Definition 1 Liu and Liu, 2003. Let (Ω, Σ, Pr) be a probability space, and $(\Gamma, \mathcal{A}, Cr)$ be a credibility space. If $\xi(\omega, \gamma)$ is a map defined on $\Omega \times \Gamma$ such that for each fixed $\omega \in \Omega$, $\xi_\omega(\gamma)$ as a function of γ is a fuzzy variable defined on the credibility space, $\gamma \in \Gamma$, then we call $\xi(\omega, \gamma)$ a fuzzy random variable.

Furthermore, if $\xi_k(\omega_k, \gamma_k)$, $k = 1, 2, \dots, K$, are fuzzy random variables, then $\xi(\omega, \gamma) = (\xi_1(\omega_1, \gamma_1), \xi_2(\omega_2, \gamma_2), \dots, \xi_K(\omega_K, \gamma_K))^T$ is called a K -ary fuzzy random vector.

Definition 2 Liu and Liu, 2003. Let $\xi(\omega, \gamma)$ be a fuzzy random variable, and B a Borel subset of \mathfrak{R} . Then the equilibrium measure Ch of an event $\{\xi(\omega, \gamma) \in B\}$ is defined as

$$Ch\{\xi(\omega, \gamma) \in B\} = \bigvee_{0 < \alpha \leq 1} [\alpha \wedge Pr\{\omega \in \Omega | Cr\{\gamma \in \Gamma | \xi_\omega(\gamma) \in B\} \geq \alpha\}].$$

Definition 3 Liu and Liu, 2003. Let $\xi(\omega, \gamma)$ be a fuzzy random variable. The expected value $E[\xi(\omega, \gamma)]$ of $\xi(\omega, \gamma)$ is defined as

$$E[\xi(\omega, \gamma)] = \int_0^{+\infty} Ch\{\xi(\omega, \gamma) \geq r\} dr - \int_{-\infty}^0 Ch\{\xi(\omega, \gamma) \leq r\} dr.$$

In addition, Liu and Liu (2005b) stated that the expected value $E[\xi(\omega, \gamma)]$ can be rewritten as the following equivalent form

$$E[\xi(\omega, \gamma)] = \int_0^{+\infty} Pr\{\omega \in \Omega | E[\xi_\omega] \geq r\} dr - \int_{-\infty}^0 Pr\{\omega \in \Omega | E[\xi_\omega] \leq r\} dr,$$

where $E[\xi_\omega]$ represents for the expected value of the fuzzy variable ξ_ω , defined by Liu and Liu (2002) as

$$E[\xi_\omega] = \int_0^{+\infty} Cr\{\xi_\omega \geq r\} dr - \int_{-\infty}^0 Cr\{\xi_\omega \leq r\} dr.$$

Definition 4 Liu and Liu, 2003. Let $\xi(\omega, \gamma)$ be a fuzzy random variable. The fuzzy random critical value of $\xi(\omega, \gamma)$ with equilibrium level α is expressed as the following form:

$$CV_\alpha(\xi(\omega, \gamma)) = \min\{r | \text{Ch}\{\xi(\omega, \gamma) \leq r\} \geq \alpha\},$$

which is equivalent to

$$CV_\alpha(\xi(\omega, \gamma)) = \min\{r | \Pr\{\omega \in \Omega | \text{Cr}\{\xi_\omega \leq r\} \geq \alpha\} \geq \alpha\}.$$

Here, the level α is taken twice to represent different meanings. That is, the first on the left represents credibility level, and the second on the right represents probability level.

In practice IHSND problem, investigations to collect data are usually conducted by experienced decision makers. However, they are unable to provide exact data about the transportation cost and travel time in advance. Instead, they describe the transportation cost (or travel time) between any two nodes using linguistic terms as an interval $[c - a, c + b]$ (or $[t - e, t + f]$) with a most possible value c (or t). Since different decision makers have different transportation cost/travel time estimations, a (or e) and b (or f) from the decision makers are selected as the left and right width of the fuzzy variable, respectively, and the most possible values c (or t) are characterized using a stochastic distribution. For example, the most possible value c (or t) can be treated as a random variable. Based on a statistical analysis of the transportation cost/travel time, estimations given by different decision makers, c (or t) can be estimated as a normal distribution. Therefore, the triangular fuzzy random number for transportation cost and travel time can be expressed as $(c - a, c, c + b)$ and $(t - e, t, t + f)$ with $c \sim \mathcal{N}(\mu_c, \rho_c^2)$ and $t \sim \mathcal{N}(\mu_t, \rho_t^2)$, respectively. The fuzzy random transportation cost and travel time representation process proposed in this research is shown in Fig. 2. In short, a fuzzy random variable is used in this study to represent the mixed uncertainty (subjective and objective) in the transportation cost and travel time.

3.2. Formulation of problem

We formulate the IHSND problem under uncertainty as an equilibrium chance program. To provide a precise statement of this problem, we define:

Indices and sets:

- $M = \{1, 2, 3\}$: the set of transportation modes, indexed by m ;
- $N = \{1, 2, \dots, n\}$: the set of all nodes, indexed by i, j, k, l ;
- i, j : the spoke nodes;
- k, l : the hub nodes.

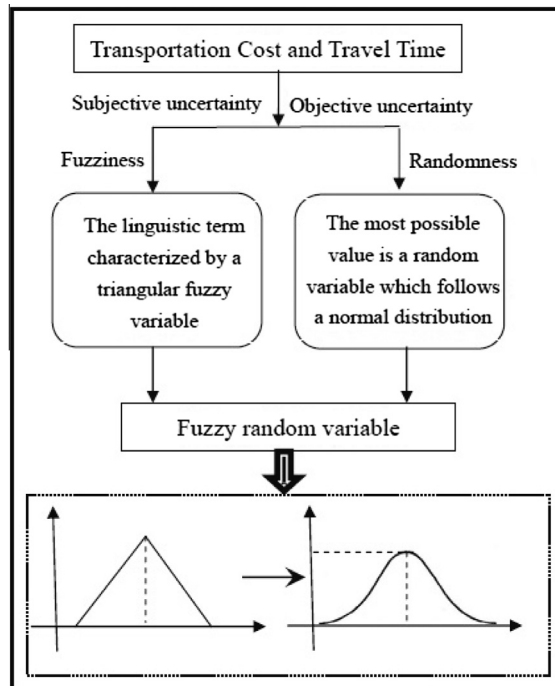


Fig. 2. The fuzzy random transportation cost and travel time representation process.

Parameters:

- d_1 : the cost discount factor on links between hubs;
- d_2 : the time discount factor on links between hubs;
- α : the predetermined equilibrium level;
- p : the number of hubs to be selected;
- \bar{T} : the maximum travel time requirement;
- $C_{ij}^m(\omega, \gamma)$: the fuzzy random transportation cost of mode m on the link from nodes i to j ;
- $T_{ij}^m(\omega, \gamma)$: the fuzzy random travel time of mode m on the link from nodes i to j .

Decision variables:

For each pair $i, k \in N$, we define the following binary decision variables,

$$Z_{ik} = \begin{cases} 1, & \text{if node } i \text{ is assigned to hub } k \\ 0, & \text{otherwise.} \end{cases}$$

When $i = k$, the variable Z_{kk} represents the establishment or not establishment of a hub at node k .

We define additional binary decision variables X_{ijkl}^m that represent path in network from node i to node j via hub k first then hub l by interhub transportation mode m , i.e.,

$$X_{ijkl}^m = \begin{cases} 1, & \text{if exists a path from node } i \text{ to } j \text{ via hub } k \text{ first then } l \text{ using transportation mode } m \\ 0, & \text{otherwise.} \end{cases}$$

Assumptions. A number of simplified assumptions are made in this paper. Specifically, it is assumed that:

- (A1) The hub network is a complete graph, i.e., every hub pair is connected via a direct link;
- (A2) There are no capacities involved;
- (A3) Direct transportation between non-hub nodes is not allowed, which means that all the traffic should be routed via at least one hub;
- (A4) Pickup from origins and drop-off to destinations are done over the road transportation.

Objective functions: In this paper, two type of decision criteria are presented for systematically modeling the IHSND problem under mixed uncertainty. Specifically, the expectation criterion is to evaluate the total transportation costs for all O-D pairs, and the critical value criterion is to quantify the service level, which can be commonly characterized by the maximum travel time requirement between O-D pairs. For simplicity, we denote a decision vector as $(\mathbf{X}, \mathbf{Z}) = \{ (X_{ijkl}^m, Z_{ik}) | \forall i, j, k, l \in N, m \in M \}$.

(1) The total transportation costs for all O-D pairs is represented as

$$C(\mathbf{X}, \mathbf{Z} | \omega, \gamma) = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \sum_{m \in M} (C_{ik}^1(\omega, \gamma) + d_1 C_{kl}^m(\omega, \gamma) + C_{lj}^1(\omega, \gamma)) X_{ijkl}^m.$$

Based on the expectation criterion, the financial-objective function can be formulated as

$$F1 = E[C(\mathbf{X}, \mathbf{Z} | \omega, \gamma)] = \int_0^{+\infty} \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | C(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \geq r\} dr - \int_{-\infty}^0 \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | C(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq r\} dr.$$

(2) The travel time requirement on a path $i \rightarrow k \rightarrow l \rightarrow j$ between O-D pairs is represented as

$$T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) = (T_{ik}^1(\omega, \gamma) + d_2 T_{kl}^m(\omega, \gamma) + T_{lj}^1(\omega, \gamma)) X_{ijkl}^m, \quad \forall i, j, k, l \in N, m \in M.$$

According to the critical value criterion, the service-objective is to minimize the critical value of the maximum travel time requirement in the sense that

$$\min \{ \bar{T} | \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha, \forall i, j, k, l \in N, m \in M \}.$$

In general, optimizing the service-objective function can be formulated as solving a fuzzy random chance-constrained programming model. As a result, it can be expressed as the minimization of the maximum travel time requirement, \bar{T} , subject to the equilibrium constraints, given below:

$$\min F2 = \bar{T} \text{ such that } \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha, \quad \forall i, j, k, l \in N, m \in M.$$

On the basis of aforementioned analysis, the proposed model minimizes a weighted sum of two objectives, $\lambda F1 + (1 - \lambda)F2$, where $0 \leq \lambda \leq 1$. The problem of IH&S network planning and optimization under mixed uncertainty can be expressed by the following fuzzy random program, which we call the fuzzy random intermodal hub-and-spoke network design (FR-IHSND) problem:

$$\text{Minimize } \lambda F1 + (1 - \lambda)F2 \quad (1)$$

$$\text{Subject to: } \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha, \quad \forall i, j, k, l \in N, m \in M \quad (2)$$

$$\sum_{m \in M} X_{ijkl}^m \geq Z_{ik} + Z_{jm} - 1, \quad \forall i, j, k, l \in N \quad (3)$$

$$\sum_{k \in N} Z_{ik} = 1, \quad \forall i \in N \quad (4)$$

$$Z_{ik} \leq Z_{kk}, \quad \forall i, k \in N \quad (5)$$

$$\sum_{k \in N} Z_{kk} = p \quad (6)$$

$$Z_{ik} \in \{0, 1\}, \quad \forall i, k \in N \quad (7)$$

$$X_{ijkl}^m \in \{0, 1\}, \quad \forall i, j, k, l \in N, m \in M. \quad (8)$$

Constraint (3) ensures that valid transportation mode assignment between each pair of located hubs. Constraint (4) imposes single assignment of nodes to hubs. Constraint (5) states that a spoke node i can only be assigned to an open hub at node k . Constraint (6) requires that exactly p hubs are established in the IH&S network. Finally, constraints (7) and (8) are binary constraints.

The FR-IHSND problem has the following unique characteristics: (i) the mode-change transshipment lines at hubs need to be designed, (ii) the multidimensional nature of IHSND problem is addressed by considering transportation cost and travel time simultaneously, and (iii) the expectation criterion and critical value criterion are presented for systematically characterizing the performance of the IH&S network. By considering these three features, we take the first initiative to investigate the FR-IHSND problem from a more realistic point of view, which contributes a new perspective to HSND theories.

3.3. Complexity of model

Next, we are particularly interested in analyzing the complexity of the proposed formulation. The following discussion will detail the total numbers of variables and constraints involved in the problem, listed in Table 1, where $|\cdot|$ represents the cardinality of a set $\{\cdot\}$. The FR-IHSND problem has $|M||N|^4 + |N|^4 + |N|^2 + |N| + 1$ constraints and $|M||N|^4 + |N|^2$ binary variables. Clearly, the complexity of this formulation is fully dependent on the size of the network and the number of the transportation mode. Because of its many variables and constraints, the FR-IHSND problem would be difficult to solve for large instances.

Observe that the FR-IHSND is a mixed-integer fuzzy random programming problem. By noting that existing studies have approved that conventional HSND model is NP-hard (Kara and Tansel, 2000). Not only the entire problem is NP-hard, even when all hubs are fixed, the allocation and routing problem in HSND is still NP-hard. The proposed FR-IHSND problem can be reduced to the conventional one if all transportation costs and travel times are always deterministic in one-mode transportation network, so it is also an NP-hard problem. To solve FR-IHSND problem, we may encounter the difficulty of calculating the expected objective and the equilibrium constraints defining problem. To overcome these difficulties, we turn the expected objective and the equilibrium constraints to their crisp equivalent ones in the next section.

4. Theoretical analysis of the proposed model

To solve the proposed FR-IHSND problem, the crux is to compute the expectation of fuzzy random variables in the objective and equilibrium chance of fuzzy random event in the constraints. Therefore, the solution methods for the FR-IHSND problem require conversion of expected objective and equilibrium constraints to their respective equivalents. However, this conversion is usually hard to perform for general fuzzy random transportation costs and travel times and only feasible in

Table 1
Number of variables and constraints in formulation FR-IHSND.

Variable or constraints	Total number at most
Z_{ik}	$ N ^2$
X_{ijkl}^m	$ M N ^4$
Equilibrium constraints (2)	$ M N ^4$
Constraints (3)–(7)	$ N ^4 + N ^2 + N + 1$

some special cases. In this section, we discuss some special cases, where the transportation costs and travel times are characterized by triangular fuzzy random variables, and turn the objective and constraints to their equivalent forms.

4.1. Computing expected total transportation cost

In this section, we consider the case that transportation costs are characterized by triangular fuzzy random variables, and derive the analytical expressions about the expected value for the total transportation costs. Applying the analytical expressions, we can transform the expected objective into its equivalent crisp form.

Theorem 1. Let the transportation cost $C_{ij}^m(\omega, \gamma)$ be the triangular fuzzy random variable such that for each $\omega \in \Omega$, $C_{ij,\omega}^m = (c_{ij}^m(\omega) - a_{ij}^m, c_{ij}^m(\omega), c_{ij}^m(\omega) + b_{ij}^m)$ are mutually independent fuzzy variables for $i, j, k, l \in N, m \in M$. Suppose the center value is $c_{ij}^m(\omega) \sim \mathcal{N}(\mu_{ij}^m, (\rho_{ij}^m)^2)$, a_{ij}^m and b_{ij}^m are left width and right width of the fuzzy variable $c_{ij}^m(\omega)$, respectively. Thus, the expected total transportation cost is calculated by the following formula:

$$E[C(\mathbf{X}, \mathbf{Z}|\omega, \gamma)] = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \sum_{m \in M} \left(\mu_{ik}^1 + d_1 \mu_{kl}^m + \mu_{ij}^1 - \frac{1}{4} (a_{ik}^1 + d_1 a_{kl}^m + a_{ij}^1) + \frac{1}{4} (b_{ik}^1 + d_1 b_{kl}^m + b_{ij}^1) \right) X_{ijkl}^m. \tag{9}$$

Proof. For each $\omega \in \Omega$, the fuzzy variable $C_{ij,\omega}^m$ has the following credibility distribution functions

$$\begin{aligned} \text{Cr}\{C_{ij,\omega}^m \geq x\} &= \begin{cases} 1 - \frac{x - c_{ij}^m(\omega) + a_{ij}^m}{2a_{ij}^m}, & \text{if } c_{ij}^m(\omega) - a_{ij}^m < x \leq c_{ij}^m(\omega) \\ \frac{c_{ij}^m(\omega) + b_{ij}^m - x}{2b_{ij}^m}, & \text{if } c_{ij}^m(\omega) < x \leq c_{ij}^m(\omega) + b_{ij}^m, \end{cases} \\ \text{Cr}\{C_{ij,\omega}^m \leq x\} &= \begin{cases} \frac{x - c_{ij}^m(\omega) + a_{ij}^m}{2a_{ij}^m}, & \text{if } c_{ij}^m(\omega) - a_{ij}^m < x \leq c_{ij}^m(\omega) \\ 1 - \frac{c_{ij}^m(\omega) + b_{ij}^m - x}{2b_{ij}^m}, & \text{if } c_{ij}^m(\omega) < x \leq c_{ij}^m(\omega) + b_{ij}^m. \end{cases} \end{aligned}$$

According to the definition of expected value operator of the fuzzy variable (Liu and Liu, 2002), we have the following computational results

$$E[C_{ij,\omega}^m] = \int_0^{+\infty} \text{Cr}\{C_{ij,\omega}^m \geq r\} dr - \int_{-\infty}^0 \text{Cr}\{C_{ij,\omega}^m \leq r\} dr = \frac{4c_{ij}^m(\omega) - a_{ij}^m + b_{ij}^m}{4}.$$

Suppose that $C_{ij,\omega}^m$ is a normal random variable $\mathcal{N}(\mu_{ij}^m, (\rho_{ij}^m)^2)$, so we have

$$C_{ij}^m(\omega, \gamma) = \frac{4\mu_{ij}^m - a_{ij}^m + b_{ij}^m}{4}.$$

Based on the linearity of expected value operator of random variable, we have

$$\begin{aligned} E[C(\mathbf{X}, \mathbf{Z}|\omega, \gamma)] &= \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \sum_{m \in M} (E[C_{ik}^1(\omega)] + d_1 E[C_{kl}^m(\omega)] + E[C_{ij}^1(\omega)]) X_{ijkl}^m \\ &= \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \sum_{m \in M} \left(\mu_{ik}^1 + d_1 \mu_{kl}^m + \mu_{ij}^1 - \frac{1}{4} (a_{ik}^1 + d_1 a_{kl}^m + a_{ij}^1) + \frac{1}{4} (b_{ik}^1 + d_1 b_{kl}^m + b_{ij}^1) \right) X_{ijkl}^m. \end{aligned}$$

The proof of theorem is complete. \square

4.2. Processing equilibrium constraints

In this subsection, we assume the uncertain travel times are characterized by triangular fuzzy random variables, and reduce the equilibrium constraints to their equivalent crisp constraints.

Theorem 2. Let the travel time $T_{ij}^m(\omega, \gamma)$ be the triangular fuzzy random variable such that for each $\omega \in \Omega$, $T_{ij,\omega}^m = (t_{ij}^m(\omega) - e_{ij}^m, t_{ij}^m(\omega), t_{ij}^m(\omega) + f_{ij}^m)$ are mutually independent fuzzy variables for $i, j, k, l \in N, m \in M$. Suppose the center value is $t_{ij}^m(\omega) \sim \mathcal{N}(\mu_{ij}^m, (\rho_{ij}^m)^2)$, e_{ij}^m and f_{ij}^m are left width and right width of the fuzzy variable $t_{ij}^m(\omega)$, respectively. Then we have

(i) If $0 < \alpha < 1/2$, then $\text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha$ is equivalent to

$$\left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} - (1 - 2\alpha)(e_{ik}^1 + d_2 e_{kl}^m + e_{lj}^1) \right) X_{ijkl}^m \leq \bar{T}. \tag{10}$$

(ii) If $1/2 \leq \alpha \leq 1$, then $\text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha$ is equivalent to

$$\left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} + (2\alpha - 1)(f_{ik}^1 + d_2 f_{kl}^m + f_{lj}^1) \right) X_{ijkl}^m \leq \bar{T}. \tag{11}$$

Proof. Since for each $\omega \in \Omega$, $T_{ik,\omega}^1, T_{kl,\omega}^m$ and $T_{lj,\omega}^1$ are mutually independent, according to the properties of triangular fuzzy variables (Liu and Gao, 2007), $T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1$ is also a triangular fuzzy variable.

We now consider the following credibility constraint for path $i \rightarrow k \rightarrow l \rightarrow j$,

$$\text{Cr}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq \alpha.$$

If $0 < \alpha \leq 1/2$, then we have

$$\text{Cr}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq \alpha \iff \text{Pos}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq 2\alpha \iff \bar{T} \geq (T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1)_{2\alpha}^L,$$

where $(T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1)_{2\alpha}^L$ is the left end point of 2α -cut of $T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1$.

The credibility constraint $\text{Cr}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq \alpha$ is equivalent to

$$t_{ik}^1(\omega) + d_2 t_{kl}^m(\omega) + t_{lj}^1(\omega) - (1 - 2\alpha)(e_{ik}^1 + d_2 e_{kl}^m + e_{lj}^1) \leq \bar{T}.$$

With the sum $t_{ik}^1(\omega) + d_2 t_{kl}^m(\omega) + t_{lj}^1(\omega) \sim N\left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1, (\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2\right)$, we can express the chance constraint $\text{Pr}\{t_{ik}^1(\omega) + d_2 t_{kl}^m(\omega) + t_{lj}^1(\omega) - (1 - 2\alpha)(e_{ik}^1 + d_2 e_{kl}^m + e_{lj}^1) \leq \bar{T}\} \geq \alpha$ as below:

$$\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} - (1 - 2\alpha)(e_{ik}^1 + d_2 e_{kl}^m + e_{lj}^1) \leq \bar{T},$$

where Z_α is the Z-value corresponding to the 100α th percentile from the standard normal distribution.

It follows that $\text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha$ can be expressed as

$$\left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} - (1 - 2\alpha)(e_{ik}^1 + d_2 e_{kl}^m + e_{lj}^1) \right) X_{ijkl}^m \leq \bar{T},$$

If $1/2 < \alpha \leq 1$, then one has

$$\text{Cr}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq \alpha \iff \text{Nec}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq 2\alpha - 1 \iff \bar{T} \geq (T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1)_{2-2\alpha}^R,$$

in which $(T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1)_{2-2\alpha}^R$ is the right end point of $(2 - 2\alpha)$ -cut of $T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1$.

The credibility constraint $\text{Cr}\{T_{ik,\omega}^1 + d_2 T_{kl,\omega}^m + T_{lj,\omega}^1 \leq \bar{T}\} \geq \alpha$ is equivalent to

$$t_{ik}^1(\omega) + d_2 t_{kl}^m(\omega) + t_{lj}^1(\omega) + (2\alpha - 1)(f_{ik}^1 + d_2 f_{kl}^m + f_{lj}^1) \leq \bar{T}.$$

Similarly, we can express the chance constraint $\text{Pr}\{t_{ik}^1(\omega) + d_2 t_{kl}^m(\omega) + t_{lj}^1(\omega) + (2\alpha - 1)(f_{ik}^1 + d_2 f_{kl}^m + f_{lj}^1) \leq \bar{T}\} \geq \alpha$ as below:

$$\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} + (2\alpha - 1)(f_{ik}^1 + d_2 f_{kl}^m + f_{lj}^1) \leq \bar{T}.$$

It follows that $\text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z} | \omega, \gamma) \leq \bar{T}\} \geq \alpha$ can be expressed as

$$\left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} + (2\alpha - 1)(f_{ik}^1 + d_2 f_{kl}^m + f_{lj}^1) \right) X_{ijkl}^m \leq \bar{T}.$$

The proof of theorem is complete. \square

4.3. Equivalent mixed-integer programming model

This subsection employs a parametric decomposition method to divide the proposed FR-IHSND problem into two equivalent crisp models. In this way, the steps of the proposed approach can be summarized as follows.

In the case of $0 < \alpha < 1/2$, according to the discussion in [Theorems 1 and 2](#), we can transform the FR-IHSND problem to the following mixed-integer programming model

$$\left\{ \begin{array}{l} \text{Minimize} \quad F1 + (1 - \lambda)F2 \\ \text{Subject to :} \quad \left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} \right. \\ \qquad \qquad \qquad \left. - (1 - 2\alpha)(e_{ik}^1 + d_2 e_{kl}^m + e_{lj}^1) \right) X_{ijkl}^m \leq \bar{T}, \quad \forall i, j, k, l \in N, m \in M \\ \text{Constraints (3)–(8),} \end{array} \right. \tag{12}$$

where $F1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \sum_{m \in M} (\mu_{ik}^1 + d_1 \mu_{kl}^m + \mu_{lj}^1 - \frac{1}{4}(a_{ik}^1 + d_1 a_{kl}^m + a_{lj}^1) + \frac{1}{4}(b_{ik}^1 + d_1 b_{kl}^m + b_{lj}^1)) X_{ijkl}^m$ and $F2 = \bar{T}$.

In the case of $1/2 \leq \alpha \leq 1$, the FR-IHSND problem can be converted into the following mixed-integer programming model

$$\left\{ \begin{array}{l} \text{Minimize} \quad F1 + (1 - \lambda)F2 \\ \text{Subject to :} \quad \left(\mu_{ik}^1 + d_2 \mu_{kl}^m + \mu_{lj}^1 + Z_\alpha \sqrt{(\rho_{ik}^1)^2 + d_2^2 (\rho_{kl}^m)^2 + (\rho_{lj}^1)^2} \right. \\ \qquad \qquad \qquad \left. + (2\alpha - 1)(f_{ik}^1 + d_2 f_{kl}^m + f_{lj}^1) \right) X_{ijkl}^m \leq \bar{T}, \quad \forall i, j, k, l \in N, m \in M \\ \text{Constraints (3)–(8),} \end{array} \right. \tag{13}$$

where $F1 = \sum_{i \in N} \sum_{j \in N} \sum_{k \in N} \sum_{l \in N} \sum_{m \in M} (\mu_{ik}^1 + d_1 \mu_{kl}^m + \mu_{lj}^1 - \frac{1}{4}(a_{ik}^1 + d_1 a_{kl}^m + a_{lj}^1) + \frac{1}{4}(b_{ik}^1 + d_1 b_{kl}^m + b_{lj}^1)) X_{ijkl}^m$ and $F2 = \bar{T}$.

Since problems (12) and (13) are mixed-integer programming problems, one possible solution method is to use a standard branch-and-bound solver. The LINGO solver, which is a state-of-the-art commercial general branch-and-bound IP-code, can be applied to deal with this problem. Furthermore, today's IP codes have become increasingly complex with the incorporation of sophisticated algorithmic components, such as advanced search strategies, preprocessing and probing techniques, cutting plane algorithms, and primal heuristics. The structure of the constraints in the problems (12) and (13) makes the use of modeling language particularly appropriate. This yields a rather efficient solution method for this kind of problem.

5. Fuzzy random simulation

In the above section, we have discussed the crisp counterpart model for the proposed FR-IHSND problem with triangular fuzzy random coefficients. However, it is still difficult to convert the general FR-IHSND problems into their deterministic counterparts. Due to the complexity, we design some fuzzy random simulations ([Liu and Liu, 2005a; Liu, 2007](#)) to calculate the uncertain functions in this paper. The first type of the uncertain function is

$$U_1 : (\mathbf{X}, \mathbf{Z}) \rightarrow \int_0^{+\infty} \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | C(\mathbf{X}, \mathbf{Z}|\omega, \gamma) \geq r\} dr - \int_{-\infty}^0 \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | C(\mathbf{X}, \mathbf{Z}|\omega, \gamma) \leq r\} dr.$$

which is the expected total transportation cost in the FR-IHSND model. In order to compute the expected cost U_1 , we design a fuzzy random simulation by integrating the fuzzy simulation and the stochastic simulation as follows.

Algorithm 1. Fuzzy random simulation for the expected value

-
- 1: Set $U_1 = 0$.
 - 2: Generate ω from Ω according to the probability measure Pr.
 - 3: Let $U_1 \leftarrow U_1 + E[C(\mathbf{X}, \mathbf{Z}|\omega)]$, where $E[C(\mathbf{X}, \mathbf{Z}|\omega)]$ can be calculated by fuzzy simulation.
 - 4: Repeat the second and third steps S times, where S is a sufficiently large integer.
 - 5: Return U_1/S .
-

The second type of the uncertain function is

$$U_2 : (\mathbf{X}, \mathbf{Z}) \rightarrow \min \{ \bar{T} | \text{Ch}\{(\omega, \gamma) \in \Omega \times \Gamma | T(\mathbf{X}, \mathbf{Z}|\omega, \gamma) \leq \bar{T}\} \geq \alpha, \forall i, j, k, l \in N, m \in M \},$$

which is the critical value of travel time requirement in the FR-IHSND model. Combining the fuzzy simulation and the stochastic simulation, we design the following fuzzy random simulation to obtain the critical value U_2 .

Algorithm 2. Fuzzy random simulation for the critical value

-
- 1: Generate $\omega_1, \omega_2, \dots, \omega_S$ from Ω according to the probability measure Pr , where S is a sufficiently large integer.
 - 2: Find the smallest values \bar{T}_s such that $\min \{\text{Cr}\{\omega \in \Omega | T(\mathbf{X}, \mathbf{Z}(\omega)) \leq \bar{T}_s\} \geq \alpha, \forall i, j, k, l \in N, m \in M\}$, for $s = 1, 2, \dots, S$, respectively, by adopting the fuzzy simulation method.
 - 3: Set S' as the integer part of $\alpha S'$.
 - 4: Return the S' th smallest element in $\{\bar{T}_1, \bar{T}_2, \dots, \bar{T}_S\}$.
-

Based on the above analysis, some insights for the method of solving the general FR-IHSND model can be obtained. Since the general FR-IHSND problem contains the general continuous fuzzy random variables for transportation cost and travel time, converting the FR-IHSND problem into the crisp counterpart model cannot be applied. In addition, as it was mentioned in the subsection 3.3, the proposed FR-IHSND problem is in general computationally intractable. For small-scale problems, branch-and-bound method can be used to obtain the optimal solution. For large-scale problems, however, using branch-and-bound method is difficult and time-consuming to get the optimal solution.

A hybrid intelligent algorithm which incorporates fuzzy random simulation technique with multi-start simulated annealing (MSA) algorithm is a good choice to overcome the difficulties mentioned above. In next section, we first briefly describe the framework of MSA algorithm, and then design a hybrid MSA algorithm by combining fuzzy random simulation technique to solve the FR-IHSND problem in this paper.

6. A hybrid multi-start simulated annealing algorithm

The SA algorithm proposed by Kirkpatrick et al. (1983) is a meta-heuristic that prevents the local search methods to quickly converge and stuck at one of the local minima. The SA search process accepts not only better solutions but also the worse neighbor solutions with a certain probability to escape from a local minimum. The probability of accepting a worse solution is determined by a parameter called temperature. The probability of accepting a worse solution is large at higher temperatures. As the temperature decreases, the probability of accepting a worse solution decreases as well. This feature can be an advantage, and has demonstrated considerable success in providing good solutions to many highly complicated integer programming problems (Azizi and Zolfaghari, 2004; El-Bouri et al., 2007; Yu and Lin, 2015).

Recall that the conventional SA algorithm is slow in convergence, and the implementation of SA algorithms is problem-dependent. To overcome such intrinsic limitations, we design a hybrid MSA algorithm to solve the proposed model. Comparing with previous works, we have made several improvements so as to better fit our problem and enhance the performance of the hybrid algorithm, including:

- (i) We propose a multi-start simulated annealing (MSA) algorithm for solving the FR-IHSND problem. MSA combines the advantages of the simulated annealing algorithm with the multi-start strategy.
- (ii) We employ a specific representation to keep the feasibility of solutions. It consists of location of the hubs, allocation of the spokes to the located hub and assignment of transportation modes.
- (iii) We adopt FRS techniques to evaluate the solutions by handling the expected objective and the general equilibrium constraints which are usually hard to be converted into their crisp equivalents.
- (iv) We design three types of transitions to generate neighbor based on the group. These transitions increase the diversity of solution so as to further enhance the search ability of hybrid algorithm.

The HMSA algorithm of solving the FR-IHSND problem is in detail described as below.

Solution representation: The solution of the FR-IHSND problem should represent the IHS network by demonstrating the location of hubs, allocation of spoke nodes to hubs and the assignment of transportation modes. In this paper, a new variant of solution representation is proposed for the proposed FR-IHSND problem to avoid the infeasible solutions through the algorithm. The proposed solution representation consists of two parts, including (a) location of the hubs and allocation of the spokes to the located hub, and (b) assignment of transportation modes. These two parts are explained as follows.

(a) The procedure for the location of hubs and allocation of spoke nodes to hubs uses an array (or string) with the length of $1 \times |N|$, where the length of the array, $|N|$, corresponds to the total number of nodes in the network (Azizi et al., 2016). Encoding the string from left to right, the first location corresponds to node number 1, the second location to node number 2, and so on. Each location on the string (i.e., a gene) contains a number which may or may not be the same as the “column number”. These numbers represent the hub facilities in the network to which one or more nodes (i.e., column number) are allocated. Each hub node is allocated to itself; this is shown where a column number matches a hub number.

As it is shown in Fig. 3, for $n = 10, p = 3$, the nodes 2, 4 and 7 are chosen as hubs. Also, the non-hub nodes 1, 3 and 5 are assigned to hub 2, the non-hub nodes 6 and 10 are assigned to hub 4 and the non-hub nodes 8 and 9 are assigned to hub 7.

(b) In order to assign the transportation modes to the hub-to-hub links, a $(p \times p)$ matrix is created by random numbers between 0 and 1. All bits of the matrix are multiplied by the number of modes and rounded up. The value of arrays corresponding to the link between each pair of hubs denotes the number of mode assigned at that link.

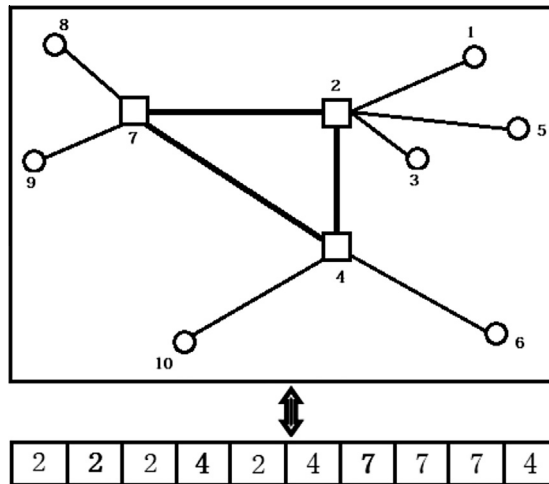


Fig. 3. Hub location and allocation representation.

Consider the above-mentioned example of Fig. 3, there are links between hubs 2–4, 2–7 and 4–7. In Fig. 4, modes number 1, 2 and 3 are assigned to the links between hubs 2–4, 2–7 and 4–7, respectively.

Initialization process: The initial solutions $X_t, t = 1, 2, \dots, P_{size}$, are generated randomly and used as multi-start points, where P_{size} denote the number of starting points in the MSA algorithm.

Evaluation by fuzzy random simulation: Calculate the objective values for all solutions $X_t, t = 1, 2, \dots, P_{size}$, by the designed fuzzy random simulations mentioned in Section 5 for computing the uncertain functions.

Neighborhood structure: The sets $N(X_t), t = 1, 2, \dots, P_{size}$, are the sets of neighboring solutions of X_t . X_t can be generated through transitions. In this work, we suggest using three types of transitions to generate neighborhood solutions. We define a group as the set of nodes allocated to the same hub. Our solution then has p groups. The employed transitions are of three types:

(T1) Change the location of the hub within a randomly chosen group to a different (randomly chosen) node in the group. This transition operator is called an exchange, which helps to solve the location problem. Fig. 5 illustrates this transition.

(T2) Change the allocation of a randomly chosen non-hub node to a different group. This transition operator is called a shift, which helps to solve the allocation problem. Fig. 6 illustrates this transition.

(T3) In the special case where a group contains only a single node, i.e. the group consists of only the hub, we perform the following transition.

- Pick a spoke node at random and make it a hub,
- Allocate the previous single hub node to a randomly chosen group.

This operator is treated as a special case of a transition (indeed it consists of two shifts and an exchange), namely hybrid operator. The hybrid operator serves to increase the diversity of solutions obtained. In particular, we find this to be beneficial in escaping local optima. Fig. 7 illustrates this transition.

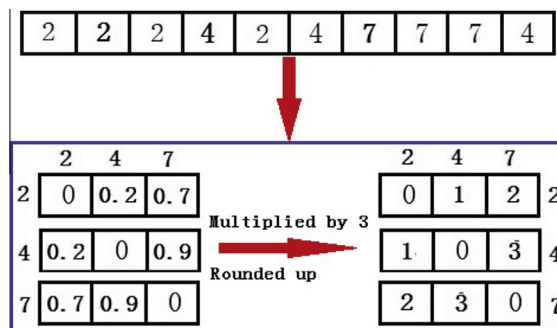


Fig. 4. Scheme of transportation modes between connected hubs.

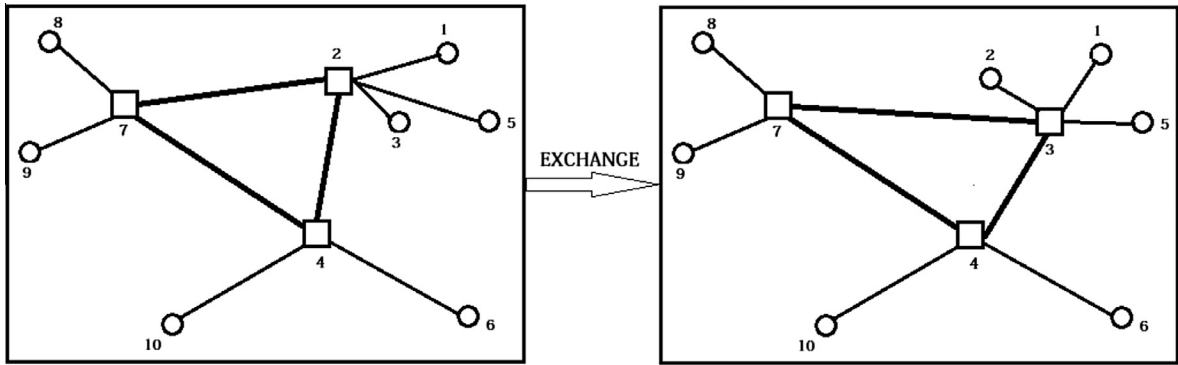


Fig. 5. An example of exchange operation.

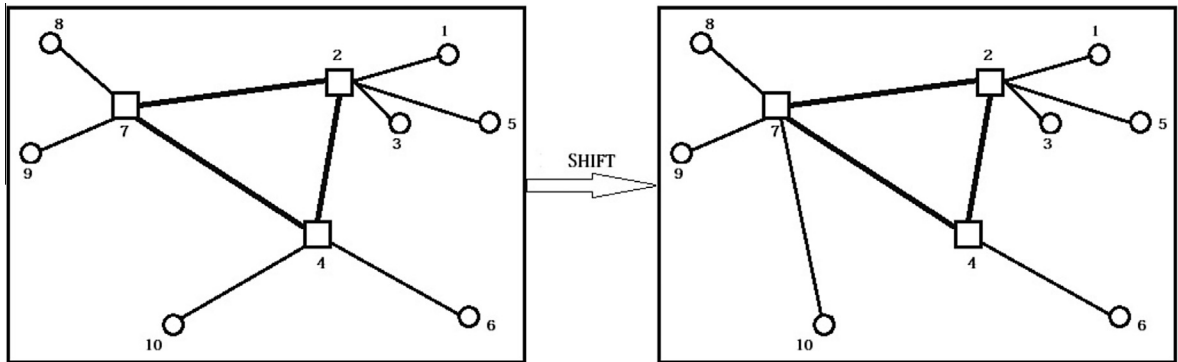


Fig. 6. An example of shift operation.

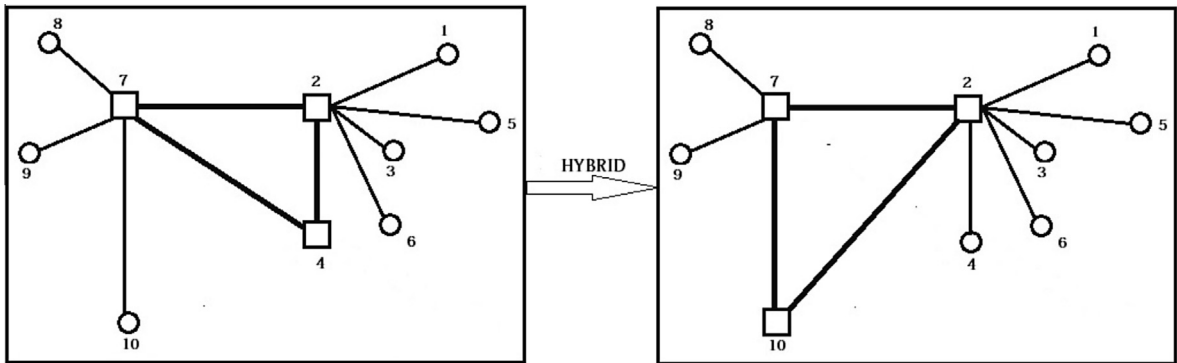


Fig. 7. An example of hybrid operation.

At each iteration, the next solution X'_t is selected from $N(X_t)$ either by exchange, shift or the hybrid operator. In order to obtain a solution in the neighborhood of the current one, we choose one of the above transition operators. The transition probabilities, defined as the probability of choosing one of the operators, are initially set at 0.6 and 0.4 for exchanges and shifts, respectively. The third transition is used when the group contains only a single node.

Acceptance/rejection of a solution: The neighborhood solution is accepted if the objective function improves, otherwise the solution is accepted with a probability depending on the temperature, which is set to allow the acceptance of a large proportion of generated solutions at the beginning (high temperature). Then, the temperature is modified (lowered) to decrease the probability of acceptance. At each temperature a predetermined number of moves are attempted. More precisely, assume that the current best solution X_t and the best objective function value obtained so far are set to be X_t and $obj(X_t)$, respectively. At each iteration, the next solution X'_t is generated from $N(X_t)$ and its objective function value is eval-

uated. Let Δ_t denote the difference between $\text{obj}(X_t)$ and $\text{obj}(X'_t)$ that is $\Delta_t = \text{obj}(X'_t) - \text{obj}(X_t)$. The probability of replacing X_t with X'_t , given that $\Delta_t > 0$ is $\exp(-\Delta_t/KT)$, where K is the Boltzmann constant used in the probability function to determine whether to accept a worse solution or not. This is accomplished by generating a random number $r \in [0, 1]$ and replacing the solution X_t with X'_t if $r < \exp(-\Delta_t/KT)$. Meanwhile, if $\Delta_t \leq 0$, the probability of replacing X_t with X'_t is 1. X_{best} records the best solution found so far as the algorithm progresses.

Cooling schedule: Selection of the simulated annealing parameters is called a cooling schedule. This gives how high the starting temperature should be, and the rules to determine it. This also gives (i) when the current temperature should be lowered, (ii) by how much the temperature should be lowered. In our simulated annealing algorithm, I_{iter} denotes the number of iterations the search proceeds at a particular temperature. T_0 represents the initial temperature. The current temperature T is decreased after running I_{iter} iterations due to the previous decrease, according to the formula $T \leftarrow vT$, where $0 < v < 1$ is the coefficient controlling the cooling schedule.

Termination: If the previously determined parameters are reached the algorithm terminates, e.g. maximum number of iterations, final temperature, no improvement for a number of iterations, etc. Our simulated annealing algorithm is terminated when the current temperature T is lower than T_F or the current best solution X_{best} is not improved in $N_{non-improving}$ consecutive temperature reductions.

Remark 1. The HMSA algorithm (without FRS technique) can be directly employed to deal with the special case in which the transportation costs and travel times are characterized by triangular fuzzy random variables.

7. Numerical experiments

In this section, two sets of numerical experiments, involving a small-scale case (denoted as RG dataset for simplicity) and a real-world instance based on the Australia Postal (AP) dataset are conducted to validate the performance of the proposed model and the performance of the proposed approach.

The HMSA parameters are presented as follows. The population size P_{size} for the small-scale experiment and large-scale experiment is set to 2 and 5, respectively. Other parameters used in the experiment are: $v = 0.970, I_{iter} = 100, K = 0.3, T_0 = 1000, T_F = 1$ and $N_{non-improving} = 50$. The computational time for the RG dataset and AP dataset is set to 10 and 100s, respectively. Each test problem runs 10 times and the best results are reported. The proposed HMSA algorithm has been coded in C++ programming language. All numerical tests were carried out on a personal computer (Lenovo with Intel(R) Core(TM) i3-4170 CPU @ 3.70GHZ and RAM 4.00 GB), using the Microsoft Windows 7 operating system.

7.1. Experiments in a small-scale network

In this section, we randomly generate a small-scale data set. For this small-scale model, the proposed FR-IHSND is converted into a deterministic programming problem where the transportation costs and the travel times are characterized by triangular fuzzy random variables. In this case, we directly solve crisp counterpart model by a Branch and Bound solver of a general-purpose optimization software, LINGO.

In this RG dataset, road, rail, and air transportation modes have been considered. We randomly generate 10 nodes data set on the plane. For each node pair $i, j = 1, 2, \dots, 10$, we assume that triangular fuzzy random transportation cost and travel time for road transportation mode are characterized by $C_{ij,\omega}^1 = (c_{ij}^1(\omega) - 3, c_{ij}^1(\omega), c_{ij}^1(\omega) + 1)$ and $T_{ij,\omega}^1 = (t_{ij}^1(\omega) - 1, t_{ij}^1(\omega), t_{ij}^1(\omega) + 1)$, where $c_{ij}^1(\omega)$ and $t_{ij}^1(\omega)$ are listed in Tables 2 and 3, respectively. In order to generate the rail and air transportation costs, an experimental parameter, cost ratio (CR) is used. We observe that the cost value

Table 2
The data set about c_{ij}^1 from node i to node j .

	1	2	3	4	5	6	7	8	9	10
1	0									
2	$\mathcal{N}(17, 2^2)$	0								
3	$\mathcal{N}(7, 1)$	$\mathcal{N}(21, 2^2)$	0							
4	$\mathcal{N}(22, 2^2)$	$\mathcal{N}(30, 3^2)$	$\mathcal{N}(16, 2^2)$	0						
5	$\mathcal{N}(38, 3^2)$	$\mathcal{N}(48, 4^2)$	$\mathcal{N}(31, 3^2)$	$\mathcal{N}(19, 1)$	0					
6	$\mathcal{N}(44, 4^2)$	$\mathcal{N}(47, 4^2)$	$\mathcal{N}(38, 3^2)$	$\mathcal{N}(22, 2^2)$	$\mathcal{N}(18, 2^2)$	0				
7	$\mathcal{N}(84, 8^2)$	$\mathcal{N}(94, 9^2)$	$\mathcal{N}(77, 7^2)$	$\mathcal{N}(66, 6^2)$	$\mathcal{N}(47, 4^2)$	$\mathcal{N}(55, 5^2)$	0			
8	$\mathcal{N}(67, 6^2)$	$\mathcal{N}(76, 7^2)$	$\mathcal{N}(60, 6^2)$	$\mathcal{N}(46, 4^2)$	$\mathcal{N}(28, 2^2)$	$\mathcal{N}(32, 3^2)$	$\mathcal{N}(25, 2^2)$	0		
9	$\mathcal{N}(56, 5^2)$	$\mathcal{N}(44, 4^2)$	$\mathcal{N}(56, 5^2)$	$\mathcal{N}(50, 5^2)$	$\mathcal{N}(63, 6^2)$	$\mathcal{N}(49, 4^2)$	$\mathcal{N}(104, 10^2)$	$\mathcal{N}(79, 7^2)$	0	
10	$\mathcal{N}(75, 7^2)$	$\mathcal{N}(62, 6^2)$	$\mathcal{N}(75, 7^2)$	$\mathcal{N}(68, 6^2)$	$\mathcal{N}(80, 8^2)$	$\mathcal{N}(65, 6^2)$	$\mathcal{N}(118, 10^2)$	$\mathcal{N}(94, 9^2)$	$\mathcal{N}(19, 2^2)$	0

Table 3
The data set about t_{ij}^1 from node i to node j .

	1	2	3	4	5	6	7	8	9	10
1	0									
2	$\mathcal{N}(9, 1)$	0								
3	$\mathcal{N}(4, 1)$	$\mathcal{N}(11, 1)$	0							
4	$\mathcal{N}(11, 1)$	$\mathcal{N}(15, 1)$	$\mathcal{N}(8, 1)$	0						
5	$\mathcal{N}(19, 1)$	$\mathcal{N}(24, 1)$	$\mathcal{N}(16, 1)$	$\mathcal{N}(10, 1)$	0					
6	$\mathcal{N}(22, 1)$	$\mathcal{N}(24, 1)$	$\mathcal{N}(19, 1)$	$\mathcal{N}(11, 1)$	$\mathcal{N}(9, 1)$	0				
7	$\mathcal{N}(42, 1)$	$\mathcal{N}(47, 1)$	$\mathcal{N}(39, 1)$	$\mathcal{N}(33, 1)$	$\mathcal{N}(24, 1)$	$\mathcal{N}(28, 1)$	0			
8	$\mathcal{N}(34, 1)$	$\mathcal{N}(38, 1)$	$\mathcal{N}(30, 1)$	$\mathcal{N}(23, 1)$	$\mathcal{N}(14, 1)$	$\mathcal{N}(16, 1)$	$\mathcal{N}(13, 1)$	0		
9	$\mathcal{N}(28, 1)$	$\mathcal{N}(22, 1)$	$\mathcal{N}(28, 1)$	$\mathcal{N}(25, 1)$	$\mathcal{N}(32, 1)$	$\mathcal{N}(25, 1)$	$\mathcal{N}(52, 1)$	$\mathcal{N}(40, 1)$	0	
10	$\mathcal{N}(38, 1)$	$\mathcal{N}(31, 1)$	$\mathcal{N}(38, 1)$	$\mathcal{N}(34, 1)$	$\mathcal{N}(40, 1)$	$\mathcal{N}(33, 1)$	$\mathcal{N}(59, 1)$	$\mathcal{N}(47, 1)$	$\mathcal{N}(10, 1)$	0

associated with rail transportation is lower than that of road transportation, while the air transportation cost is higher, as expected. The rail transportation costs are generated by multiplying the road transportation costs by CR = 0.8 and the air transportation costs are generated by multiplying the road transportation costs by CR = 2.0. Similarly, travel times using rail and air transportation are calculated by the time ratio (TR). Since the travel times for using rail transportation is longer than that of road transportation, while the air travel time is shorter, we take TR = 1.2 and TR = 0.5 for rail and air transportations, respectively. The problem instances are generated by setting the number of hubs p to be opened to 3 and 5; the objective weight λ to 0.1, 0.5 and 0.9; the inter-hub discount factor $d = d_1 = d_2$ to 0.2 and 0.6; and the equilibrium level parameters α to 0.85, 0.90 and 0.95.

The meanings of the column headings are as follows: The first column gives the objective weight λ ; column 2 presents the discount factor d ; column 3 shows the different equilibrium level α ; The columns under “HMSA” and “LINGO” report the two objective function values and the average CPU time requirement in seconds. The optimality gap is calculated as $\frac{f_{HMSA} - f^*}{f^*}$ where f_{HMSA} is the value of the HMSA solution and f^* is the value of the optimal solution. The computational results of the proposed HMSA and LINGO for the RG data set are presented in Tables 4 and 5. It can be seen from Tables 4 and 5 that the cost-objective function decreases as the objective weight value increases, while the service-objective function takes an opposite tendency with respect to weights. This is an expected result because the minimization of the expected total transportation costs has relatively more priority for the high objective weight value. It is also observed from Tables 4 and 5 that the service-objective function increases as the equilibrium level increases. It is intuitive that as the equilibrium level parameter gets higher, the decision maker would set a longer travel time requirement to meet a wide range of customers satisfaction. On the other hand, as the travel time requirement increases, the cost-objective function decreases. Mainly because when

Table 4
Computational results for RG data set with $p = 3$.

λ	d	α	HMSA			LINGO			%GAP	
			Obj-cost	Obj-time	CPU (s)	Obj-cost	Obj-time	CPU (s)		
0.1	0.2	0.85	3521.77	30.26	1.3	3521.77	30.26	3700	0.00	
		0.90	3511.29	33.41	1.1	3511.29	33.41	3721	0.00	
		0.95	3502.64	36.04	1.4	3502.64	36.04	3618	0.00	
	0.6	0.85	4694.01	45.81	1.7	4694.01	45.81	3576	0.00	
		0.90	4680.92	47.62	1.5	4680.92	47.62	3613	0.00	
		0.95	4666.56	49.13	1.2	4666.56	49.13	3599	0.00	
	0.5	0.2	0.85	3517.98	31.75	1.3	3517.98	31.75	3721	0.00
			0.90	3507.37	34.42	1.1	3507.37	34.42	3698	0.00
			0.95	3500.45	37.02	1.5	3500.45	37.02	3645	0.00
0.6		0.85	4680.53	46.32	1.4	4680.53	46.32	3627	0.00	
		0.90	4666.43	48.03	1.1	4666.43	48.03	3533	0.00	
		0.95	4650.68	50.42	1.3	4650.68	50.42	3396	0.00	
0.9	0.2	0.85	3514.78	32.77	1.1	3514.78	32.77	3611	0.00	
		0.90	3503.67	35.63	1.3	3503.67	35.63	3583	0.00	
		0.95	3494.33	38.01	1.5	3494.33	38.01	3567	0.00	
	0.6	0.85	4666.44	47.13	1.0	4666.44	47.13	3701	0.00	
		0.90	4651.39	49.65	1.6	4651.39	49.65	3543	0.00	
		0.95	4636.23	51.99	1.1	4636.23	51.99	3672	0.00	
	Average					1.3	3618			

Table 5
Computational results for RG data set with $p = 5$.

λ	d	α	HMSA			LINGO			%GAP
			Obj-cost	Obj-time	CPU (s)	Obj-cost	Obj-time	CPU (s)	
0.1	0.2	0.85	2309.90	29.43	2.9	2309.90	29.43	4902	0.00
		0.90	2300.42	30.26	3.1	2300.42	30.26	4865	0.00
		0.95	2291.33	31.02	3.2	2291.33	31.02	4303	0.00
	0.6	0.85	3869.26	43.87	3.5	3869.26	43.87	4054	0.00
		0.90	3848.67	45.66	3.7	3848.67	45.66	4176	0.00
		0.95	3827.73	47.02	4.0	3827.73	47.02	3762	0.00
0.5	0.2	0.85	2307.68	30.55	3.1	2307.68	30.55	3859	0.00
		0.90	2296.34	31.43	3.3	2296.34	31.43	3911	0.00
		0.95	2285.98	32.02	3.0	2285.98	32.02	3865	0.00
	0.6	0.85	3856.73	44.98	3.3	3856.73	44.98	4532	0.00
		0.90	3835.22	47.65	4.0	3835.22	47.65	4978	0.00
		0.95	3813.68	49.31	4.3	3813.68	49.31	5011	0.00
0.9	0.2	0.85	2305.14	31.73	3.2	2305.14	31.73	3589	0.00
		0.90	2295.33	32.68	3.5	2295.33	32.68	3643	0.00
		0.95	2284.67	33.76	3.4	2284.67	33.76	3512	0.00
	0.6	0.85	3843.28	48.85	3.6	3843.28	48.85	5473	0.00
		0.90	3824.32	50.03	3.9	3824.32	50.03	5193	0.00
		0.95	3803.76	51.99	3.2	3803.76	51.99	5202	0.00
Average					3.5	4157			

extending travel time requirement, the decision maker would prefer adopting the mode of rail transportation to meet the design requirements for the IH&S network at a lower transportation cost.

Using the Branch and Bound solver of LINGO, we solved all 36 benchmark problems to optimality. The computational results show that the proposed HMSA method surpasses the LINGO solver with respect to efficiency. It obtains optimal values in significantly less time than the LINGO solver for the same test problems. Specifically, when the problem size is 10, the average running time of HMSA method is 2.4 s. However, the LINGO solver achieves the same optimum values by an average running time of 3887.5 s. In addition, the LINGO solver fails to efficiently solve large and complicated instances. It is due to the fact that the LINGO solver achieves its optimal solution using the branch and bound procedure by examining nodes, where number of nodes become very large when the test problem size increases.

After implementing the proposed model and solution approach on the RG dataset by $\lambda = 0.5, d = 3, \alpha = 0.85$ and $p = 3$, the IH&S network structure is illustrated in Fig. 8. It can be seen that the place of hubs has been dispersed equitably through different nodes in order to respect the transportation cost and the maximum travel time requirement. Furthermore, Fig. 8 shows that nodes number 2, 4 and 7 have been considered to be as hubs. In addition, all three modes of transportation have been utilized between hubs number 2 and 7, while road and rail has been constructed from hub 4 to 7 and finally road and air ways are used from hub 2 to hub 4.

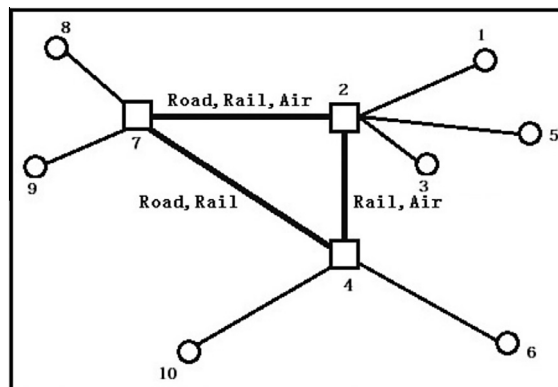


Fig. 8. IH&S network topography for $p = 3$.

7.2. Experiments in a large-scale AP dataset

In this section, we apply our approach to an application case study on a large-scale AP data set which was introduced by Ernst and Krishnamoorthy (1996) and considered to be a benchmark by most researchers in the H&S network area. By downloading the files corresponding to AP data set from the OR-Library (Beasley, 2014), we obtain the x and y coordinates of 100 cities in Australia. We select the problem sizes $N = \{50, 70, 100\}$ from the original AP data set. This data set was modified to generate data for solving the proposed FR-IHSND problem through using the following changes. For each node pair $i, j \in N$, we assume that fuzzy random transportation cost for road transportation mode is characterized by $C_{ij,\omega}^1 = (c_{ij}^1(\omega) - 1, c_{ij}^1(\omega), c_{ij}^1(\omega) + 1)$ with $c_{ij}^1(\omega) = \rho w_{ij} D_{ij}$, where w_{ij} is the deterministic flow volume, D_{ij} is the Euclidean distance given in the AP data set and ρ is randomly generated from the interval $[0, 1]$ using a uniform distribution. This range is used for computational purposes to reflect the stochastic uncertainty in transportation costs. The fuzzy random travel time between cities i and j for road transportation mode is described as $T_{ij,\omega}^1 = (t_{ij}^1(\omega) - 1, t_{ij}^1(\omega), t_{ij}^1(\omega) + 1)$, where t_{ij}^1 is sampled from an Uniform(10,100) distribution for travel time data (not given in the AP data set). The reasoning behind the choice of this interval has to do with the need to obtain data with some relevant degree of travel time variability. In addition, we generate the fuzzy random transportation cost and travel time for the rail and air transportation mode according to Section 7.1.

The proposed FR-IHSND problem with general continuous fuzzy random vectors for transportation cost and travel time of this large-scale case is difficult to be converted into its crisp equivalent. Thus, we apply the HMSA algorithm to solve it. In order to further evaluate the performance of our proposed HMSA algorithm for the FR-IHSND problem, we design another hybrid approach, that is a hybrid GA (HGA) by integrating the fuzzy random simulation method. Then we compare the experimental results of these two different approaches. In HGA procedure, we use the solution representation and initialize process which are described in Section 4.2, ensuring a fair comparison. We apply roulette wheel for fitness selection. The population size is set as 40 and the maximum generation is set as 2000. In addition, the best mutation and crossover rates (P_m, P_c) are selected for HGA after a number of experiments, where $(P_m, P_c) = (0.2, 0.3)$ for equilibrium level 0.85, $(P_m, P_c) = (0.3, 0.3)$ for equilibrium level 0.90, $(P_m, P_c) = (0.4, 0.3)$ for equilibrium level 0.95. The computational time of the HGA for the AP dataset is set to 100s. The HGA is also run on Lenovo PC by using C++ programming language. Furthermore, both two hybrid algorithms run with 1000×1000 cycles in fuzzy random simulation. Applying the different approaches to the FR-IHSND problem, we obtain the comparison results in Table 6. Comparison results show that the proposed HMSA algorithm outperforms the HGA approach in terms of solution quality when dealing with the large-scale FR-IHSND problem.

7.3. HMSA performance on the fuzzy p -hub center problem

The proposed FR-IHSND model can be easily tailored for fuzzy p -hub center problem (Yang et al., 2013) by setting the objective function weight to zero (i.e., $\lambda = 0$) and ignoring the randomness of travel times in one-mode transportation network. A number of problem instances derived form are used as a platform to evaluate the performance of proposed HMSA on this special case.

Table 7 summarizes the computational results for the fuzzy p -hub center problem. The parameters of the proposed HMSA used in this special case are: $P_{size} = 2, v = 0.970, I_{iter} = 100, K = 0.3, T_0 = 1000, T_F = 1$ and $N_{non-improving} = 50$. Computational time for each problem is fixed at 5 s. It is worth noting that the proposed HMSA solves all 36 instances to optimality in very short computing time. Therefore, our proposed method is quite general and will be applicable to other HSND problems.

7.4. Managerial insights

The computational results were analyzed above to illustrate that both the proposed model and solution method are acceptable. These days, the intermodal transportation has received increasing attention by the freight companies. The competitive marketplace requires an efficient and effective delivery strategy. Such a strategy aims at managing shipments across geographically dispersed supply and demand areas within a competitive transportation cost and a reasonable travel time. To fulfill those needs, the FR-IHSND model can offer a viable option. From the computational results, we conclude that the proposed FR-IHSND problem can provide a real representation of IH&S network.

8. Conclusion and future work

In this paper, we have addressed an IH&S network design problem under mixed uncertainty, where the transportation cost and the travel time are characterized by fuzzy random variables with credibility and probability distributions. We developed a fuzzy random programming method as a decision-making tool to model this problem. In the case where the transportation cost and the travel time are characterized by triangular fuzzy random variables, we derived the crisp counterpart model for the proposed problem. In a more general where the transportation cost and the travel time are described as

Table 6
Computational results for AP data set.

N	p	α	HMSA		HGA	
			Obj-cost	Obj-time	Obj-cost	Obj-time
50	3	0.85	155443.25	68.56	155443.25	68.56
		0.90	155006.38	76.34	155006.38	76.34
		0.95	154611.17	85.55	154611.17	85.55
	5	0.85	130645.85	53.76	130645.85	53.76
		0.90	130316.87	57.44	130316.87	57.44
		0.95	130008.99	61.75	130008.99	61.75
70	3	0.85	158872.65	93.68	158995.34	95.52
		0.90	158103.67	105.83	158217.68	107.01
		0.95	157542.62	116.69	157676.45	118.33
	5	0.85	133865.16	73.55	133929.36	76.93
		0.90	133016.78	79.89	133201.65	83.31
		0.95	132181.18	83.76	132407.35	86.34
100	3	0.85	159112.67	126.73	159342.65	135.73
		0.90	158654.37	137.54	158802.73	146.25
		0.95	158002.87	149.82	158211.82	158.31
	5	0.85	135654.86	100.43	135834.55	111.63
		0.90	134621.64	111.75	134864.72	122.48
		0.95	133673.21	120.83	133843.76	131.99

Table 7
Computational results for fuzzy p -hub center problem (Yang et al., 2013).

p	d	α	Optimal value	HMSA best	CPU (s)	%GAP
2	0.2	0.85	198.90	198.90	0.35	0.00
		0.90	212.47	212.47	0.20	0.00
		0.95	232.15	232.15	0.25	0.00
	0.6	0.85	222.21	222.21	0.20	0.00
		0.90	237.24	237.24	0.35	0.00
		0.95	259.05	259.05	0.15	0.00
3	0.2	0.85	139.01	139.01	0.85	0.00
		0.90	148.41	148.41	0.80	0.00
		0.95	162.06	162.06	0.65	0.00
	0.6	0.85	172.19	172.19	0.45	0.00
		0.90	183.88	183.88	0.75	0.00
		0.95	200.84	200.84	0.60	0.00
4	0.2	0.85	122.89	122.89	0.60	0.00
		0.90	131.28	131.28	0.95	0.00
		0.95	143.45	143.45	0.70	0.00
	0.6	0.85	169.24	169.24	0.95	0.00
		0.90	180.68	180.68	0.95	0.00
		0.95	197.29	197.29	0.95	0.00

general continuous fuzzy random vectors, we designed a hybrid intelligent algorithm which incorporates fuzzy random simulation with the multi-start simulated annealing algorithm to solve the general model. We conducted two sets of numerical experiments, involving a small scale case and a real-word instance based on the AP dataset to illustrate the applications and effectiveness of the proposed model and solution approach.

This paper opens up several opportunities for future research. First, this problem could be extended to include capacity restrictions which can be on the amount of flow processed at the hubs. Naturally, the problem will be harder to solve to optimality with the addition of these capacity restrictions. Thus, there will be a need to develop efficient solution algorithms in the future. Second, our model could be extended to incorporate pricing issues and market competition in IH&S network design. Third, it may be desirable to apply the proposed fuzzy random programming method for building IH&S network under the risk of hub disruption (Azizi et al., 2016). Finally, further work could be done to improve the hybrid intelligent algorithm in order to achieve more reliable solutions.

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References

- Alumur, S., Kara, B.Y., 2008. Network hub location problems: the state of the art. *Eur. J. Oper. Res.* 190, 1–21.
- Arnold, P., Peeters, D., Thomas, I., 2004. Modelling a rail/road multimodal transportation system. *Transport. Res. Part E: Logist. Transport. Rev.* 40, 255–270.
- Azizi, N., Zolfaghari, S., 2004. Adaptive temperature control for simulated annealing: a comparative study. *Comput. Oper. Res.* 31, 2439–2451.
- Azizi, N., Chauhan, S., Salhi, S., Vidyarthi, N., 2016. The impact of hub failure in hub-and-spoke networks: mathematical formations and solution techniques. *Comput. Oper. Res.* 65, 175–188.
- Beasley, J.E., 2014. OR-Library. <<http://people.brunel.ac.uk/mastjib/jeb/info.html>>.
- Bontekoning, Y.M., Macharis, C., Trip, J.J., 2004. Is a new applied transportation research field emerging? A review of intermodal rail truck freight transport literature. *Transport. Res. Part A* 38, 1–34.
- Calik, H., Alumur, S., Kara, B., Karasan, O., 2009. A tabu-search based heuristic for the hub covering problem over incomplete hub networks. *Comput. Oper. Res.* 36, 3088–3096.
- Campbell, J.F., 1994. Integer programming formulations of discrete hub location problems. *Eur. J. Oper. Res.* 72, 387–405.
- Chen, J.F., 2007. A hybrid heuristic for the uncapacitated single allocation hub location problem. *Omega* 35 (2), 211–220.
- Chou, C., 2010. Application of FMCDM model to selecting the hub location in the marine transportation: a case study in southeastern Asia. *Math. Comput. Model.* 51, 791–801.
- Contreras, I., Cordeau, J.F., Laporte, G., 2011. Stochastic uncapacitated hub location. *Eur. J. Oper. Res.* 212, 518–528.
- Crainic, T.G., Kim, K.H., 2007. Multimodal transportation. In: Barnhart, C., Laporte, G. (Eds.), *Transportation. Handbooks in Operations Research and Management Science*, vol. 12. North-Holland, Amsterdam, pp. 67–532.
- El-Bouri, A., Azizi, N., Zolfaghari, S., 2007. A comparative study of a new heuristic based on adaptive memory programming and simulated annealing: the case of job shop scheduling. *Eur. J. Oper. Res.* 177, 1894–1910.
- Ernst, A.T., Krishnamoorthy, M., 1996. Efficient algorithms for the uncapacitated single allocation p -hub median problem. *Locat. Sci.* 4, 139–154.
- Ernst, A.T., Krishnamoorthy, M., 1998. Exact and heuristic algorithms for the uncapacitated multiple allocation p -hub median problem. *Eur. J. Oper. Res.* 104, 100–112.
- Farahani, R.Z., Heklatfar, M., Arabani, A.B., Nikbaksh, E., 2013. Hub location problems: a review of models, classification, techniques and application. *Comput. Ind. Eng.* 64, 1096–1109.
- Gelareh, S., Monemi, R.N., Nickel, S., 2015. Multi-period hub location problems in transportation. *Transport. Res. Part E: Logist. Transport. Rev.* 75, 67–94.
- Hsiao, C.Y., Hansen, M., 2011. A passenger demand model for air transportation in a hub-and-spoke network. *Transport. Res. Part E: Logist. Transport. Rev.* 47, 1112–1125.
- Ishfaq, R., Sox, C.R., 2010. Intermodal logistics: the interplay of financial, operational and service issues. *Transport. Res. Part E: Logist. Transport. Rev.* 46, 926–949.
- Kara, B.Y., Tansel, B.C., 2000. On the single-assignment p -hub center problem. *Eur. J. Oper. Res.* 125, 648–655.
- Kwakernaak, H., 1978. Fuzzy random variables. I: Definitions and theorems. *Inform. Sci.* 15, 1–29.
- Kirkpatrick, S., Gelatt, C.D., Vecchi, M.P., 1983. Optimization by simulated annealing. *Science* 220, 671–680.
- Liu, B., Liu, Y., 2002. Expected value of fuzzy variable and fuzzy expected value models. *IEEE Trans. Fuzzy Syst.* 10, 445–450.
- Liu, Y.K., Liu, B., 2003. Fuzzy random variable: a scalar expected value operator. *Fuzzy Optimiz. Decis. Mak.* 2, 143–160.
- Liu, Y.K., Liu, B., 2005a. Fuzzy random programming with equilibrium chance constraints. *Inform. Sci.* 15, 363–395.
- Liu, Y.K., Liu, B., 2005b. On minimum-risk problems in fuzzy random decision systems. *Comput. Oper. Res.* 32, 257–283.
- Liu, Y.K., 2007. The approximation method for two-stage fuzzy random programming with recourse. *IEEE Trans. Fuzzy Syst.* 15, 1197–1208.
- Liu, Y.K., Gao, J., 2007. The independence of fuzzy variables with applications to fuzzy random optimization. *Int. J. Uncertain. Fuzz. Knowl. Syst.* 15, 1–20.
- Martí, R., Corberán, A., Peiró, J., 2015. Scatter search for an uncapacitated p -hub median problem. *Comput. Oper. Res.* 58, 53–66.
- Meng, Q., Wang, X., 2011. Intermodal hub-and-spoke network design: incorporating multiple stakeholders and multi-type containers. *Transport. Res. Part B: Methodol.* 45, 724–742.
- Mohammadi, M., Jolai, F., Tavakkoli-Moghaddam, R., 2013. Solving a new stochastic multi-mode p -hub covering location problem considering risk by a novel multi-objective algorithm. *Appl. Math. Model.* 37, 10053–10073.
- Mohammadi, M., Torabi, S.A., Tavakkoli-Moghaddam, R., 2014. Sustainable hub location under mixed uncertainty. *Transport. Res. Part E: Logist. Transport. Rev.* 62, 89–115.
- Mohammadi, M., Moghaddama, R.T., 2016. Design of a fuzzy bi-objective reliable p -hub center problem. *J. Intell. Fuzzy Syst.* 30, 2563–2580.
- O’Kelly, M.E., 1986. The location of interesting hub facilities. *Transport. Res.* 20, 92–106.
- O’Kelly, M.E., 1987. A quadratic integer program for the location of interacting hub facilities. *Eur. J. Oper. Res.* 32, 393–404.
- Racunica, I., Wynter, L., 2005. Optimal location of multimodal freight hubs. *Transport. Res. Part B* 39, 453–477.
- Randall, M., 2008. Solution approaches for the capacitated single allocation hub location problem using ant colony optimization. *Comput. Optimiz. Appl.* 39, 239–261.
- Saboury, A., Ghaffari-Nasab, N., Barzinpour, F., Jabalameli, M.S., 2013. Applying two efficient hybrid heuristics for hub location problem with fully interconnected backbone and access networks. *Comput. Oper. Res.* 40 (10), 2493–2507.
- Sim, T., Lowe, T.J., Thomas, B.W., 2009. The stochastic p -hub center problem with service-level constraints. *Comput. Oper. Res.* 36, 3166–3177.
- Skorin-Kapov, D., Skorin-Kapov, J., O’Kelly, M.E., 1996. Tight linear programming relaxations of uncapacitated p -hub median problems. *Eur. J. Oper. Res.* 94, 582–593.
- Taghipourian, F., Mahdavi, I., Mahdavi-Amiri, N., Makui, A., 2012. A fuzzy programming approach for dynamic virtual hub location problem. *Appl. Math. Model.* 36, 3257–3270.
- Yang, T.H., 2009. Stochastic air freight hub location and freight routes planning. *Appl. Math. Model.* 33, 4424–4430.
- Yang, K., Liu, Y., Yang, G., 2013. Solving fuzzy p -hub center problem by genetic algorithm incorporating local search. *Appl. Soft Comput.* 13, 2624–2632.
- Yang, K., Liu, Y., 2015. Developing equilibrium optimization methods for hub location problems. *Soft Comput.* 19, 2337–2353.
- Yu, V., Lin, S., 2015. A simulated annealing heuristic for the open location-routing problem. *Comput. Oper. Res.* 62, 184–196.